# Examiners' Report

# Summer 2015

Pearson Edexcel International Advanced Level in Further Pure Mathematics F3 (WFM03/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2015
Publications Code IA041578

**Further Pure Mathematics F3 (WFM03)**

**General introduction**

This paper proved a good test of students' knowledge and students' understanding of F3 material. There were plenty of easily accessible marks available for students who were competent in topics such as hyperbolic functions, integration, vector methods, eigenvalues and eigenvectors and coordinate geometry. Therefore, a typical E grade student had enough opportunity to gain marks across the majority of questions. At the other end of the scale, there was sufficient material, particularly in later questions to stretch and challenge the most able students.

## Report on Individual Questions

### Question 1

The vast majority of students correctly used the identity $\cosh 2x = 1 + 2\sinh^2 x$ to obtain a quadratic in sinh$x$. Most then used the logarithmic form of arsinh to obtain the final answers. Some students wrote sinh$x$ in terms of exponentials and proceeded to solve the resulting quadratics in $e^x$ and sometimes ended up with extra solutions that were not rejected. A significant number of students attempted to solve the given equation by expressing it in terms of exponentials. Such solutions usually stopped once a quartic in $e^x$ was reached. Quite often, students who adopted this approach, realised that any progress would be difficult and so resorted to using the identity $\cosh 2x = 1 + 2\sinh^2 x$.

### Question 2

This was a good source of 5 marks for many students. In part (a), almost all students used the correct eccentricity formula and substituted $x = 12$ and $y = 5$ into the hyperbola and solved the resulting simultaneous equations in $a$ and $b$. There were occasional processing errors and some gave the positive and negative values for $a$ and $b$. The majority of students knew how to find the foci in part (b) and a follow through mark was available for those with earlier errors. A significant number of students gave only the positive focus.

### Question 3

In part (a), the vast majority of students knew what an eigenvector was, and could easily establish the eigenvalue for the given eigenvector. Students could then easily show that $k$ had the value given in part (b) by using the $y$ component.

In part (c) most students made a sound attempt at the characteristic equation although there were some sign errors and occasionally some missing terms.

Part (d) was solved by one of two methods. Some students attempted $\mathbf{M}^{-1}$ and then used $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$, others set up 3 equations in 3 unknowns from $\mathbf{M}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$. The simultaneous equations method was probably the more popular of the two but both were equally successful with any error generally being arithmetic slips. A small minority of students erroneously attempted $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{M}\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$.

## Question 4

Many correct proofs were seen in part (a). The most common approach was to write $\cosh^n x$ as $\cosh^{n-1} x \cosh x$ and then attempt parts although a significant number of students differentiated $\cosh^{n-1} x$ as $(n-1)\cosh^{n-2} x$ and potentially lost a significant number of subsequent marks. Some students wrote $\cosh^n x$ as $\cosh^{n-2} x \cosh^2 x$ and then used $\cosh^2 x = 1 + \sinh^2 x$ and proceeded correctly. A significant minority of students incorrectly attempted parts using $\frac{\mathrm{d}v}{\mathrm{d}x} = 1$ and some students failed to give a convincing proof because one or more vital steps were missing.

Students often applied the reduction formula well in part (b) although there were some arithmetic slips and very occasionally the reduction formula was applied as $I_n = \sinh x \cosh^{n-1} x + (n-1) I_{n-2}$ rather than $nI_n = \sinh x \cosh^{n-1} x + (n-1) I_{n-2}$.

## Question 5

In part (a) the majority of students substituted the general equation of the straight line into the equation of the ellipse and then used the property of equal roots for the resulting quadratic in *x*. The second most popular method was to find a general tangent to the ellipse using the parametric form $(5\cos\theta, 3\sin\theta)$ to give $y = -\frac{3\cos\theta}{5\sin\theta}x + \frac{3}{\sin\theta}$ and then to show $\left(\frac{3}{\sin\theta}\right)^2 - 25\left(-\frac{3\cos\theta}{5\sin\theta}\right)^2 = 9$. A small number of students took more laborious routes involving implicit differentiation and simultaneous equations and such methods were met with varying degrees of success. It is worth pointing out here that students cannot simply quote the general result $a^2m^2 + b^2 = c^2$ and expect to gain any marks. This was, after all, the result they were being asked to prove.

Many fully correct solutions were seen in part (b) and the main method was to substitute the given point into the general straight line and then solve this simultaneously with the result achieved in part (a). There were a large number of students who thought the point (3, 4) was on the ellipse and proceeded to find the equation of the tangent to the ellipse at that point.

**Question 6**

Part (a) was potentially a source of 5 easy marks for many students. The majority found correct derivatives although there were some sign and coefficient errors. Those with correct derivatives could usually establish the printed result although quite a few got lost with the trigonometry or made minor slips.
Success in part (b) was varied and it was quite often the case that once students had reached $S = 2\pi\int(2\sin\theta - \sin 2\theta)\sqrt{8(1-\cos\theta)}\,\mathrm{d}\theta,$ they could not see how to proceed.

Students often split the integral into two and made failed attempts to integrate by parts. Those who eliminated the double angle often made further progress and there were other successful methods employed such as the use of half angles and integration by substitution. This part discriminated well.

**Question 7**

There were quite a lot of students who did not attempt this question and there were few fully correct solutions. There were many successful attempts at part (a) but some did not find a second vector in the plane in order to find the vector product. Students frequently extracted 2**i** – **j** + 4**k** from the line but used 3**i** + 3**j** – 2**k** as the second vector. A small number of students attempted a verification approach, showing that both the point and the line satisfied the Cartesian equation of the plane.

Students who attempted part (b) seemed to know the method but there were errors finding the distance between the two planes. Some found the distance from each plane to the origin and subtracted to obtain one correct solution. However many then said that the two planes were on opposite sides of the origin and added the two distances obtaining a second, incorrect solution. Many initially used the modulus and got to $\left|\frac{3\alpha-1}{5\sqrt{5}}\right| = \frac{1}{\sqrt{5}}$ but then abandoned the modulus signs and only obtained one solution and others never used the modulus, only considering the positive distance. It was relatively rare to see a solution that correctly obtained both values of $\alpha$.

**Question 8**

Many students scored well on this question. In part (a), the substitution of $x = \frac{3}{4}\sinh u$ was usually sound and a fully correct substitution was frequently seen. Attempts to simplify were also usually accurate although some students struggled to simplify $\sqrt{9 + 16 \times \frac{9}{16}\sinh^2 u}$ correctly. Many students could obtain an integrand that was a multiple of $\sinh^2 u$ but sometimes lost the factor of ½ when using the double angle identity.

Many students could make progress in part (b) with or without a correct value for *k* and could obtain an expression of the required form. The correct *u* limits were frequently seen although some students used *x* limits throughout.

Grade Boundaries

Grade boundaries for this, and all other papers, can be found on the website on this link:
http://www.edexcel.com/iwantto/Pages/grade-boundaries.aspx

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom